

Introduction to Trigonometry

1. Meaning (Definition) of Trigonometry

The word trigonometry is derived from the Greek words 'tri' meaning three, 'gon' meaning sides and 'metron' meaning measure.

Trigonometry is the study of relationships between the sides and the angles of the triangle.

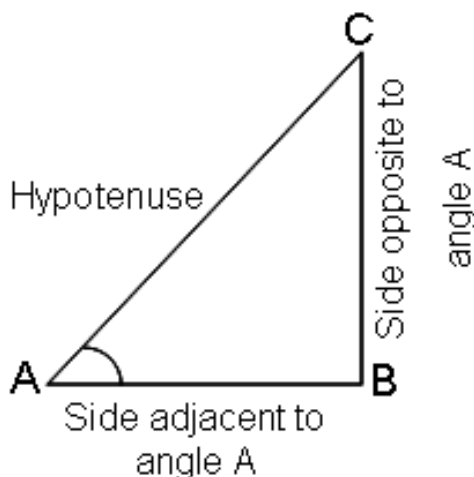
2. Positive and negative angles

Angle measured in anticlockwise direction is taken as positive angle whereas the angle measured in clockwise direction is taken as negative angle.

3. Trigonometric Ratios

Ratio of the sides of a right triangle with respect to the acute angles is called the **trigonometric ratios** of the angle.

Trigonometric ratios of the acute angle A in right triangle ABC are given as follows:



$$\begin{aligned}
 \text{i. } \sin \angle A &= \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{p}{h} \\
 \text{ii. } \cos \angle A &= \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{b}{h} \\
 \text{iii. } \tan \angle A &= \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \frac{BC}{AB} = \frac{p}{b} \\
 \text{iv. } \operatorname{cosec} \angle A &= \frac{\text{hypotenuse}}{\text{side opposite to } \angle A} = \frac{AC}{BC} = \frac{h}{p} \\
 \text{v. } \sec \angle A &= \frac{\text{hypotenuse}}{\text{side adjacent to } \angle A} = \frac{AC}{AB} = \frac{h}{b} \\
 \text{vi. } \cot \angle A &= \frac{\text{side adjacent to } \angle A}{\text{side opposite to } \angle A} = \frac{AB}{BC} = \frac{b}{p}
 \end{aligned}$$

4. Important facts about Trigonometric ratios

- Trigonometric ratios of an acute angle in a right triangle represents the relation between the angle and the sides.
- The ratios defined above can be rewritten as $\sin A$, $\cos A$, $\tan A$, $\operatorname{cosec} A$, $\sec A$ and $\cot A$.
- Each trigonometric ratio is a real number and it has not unit.

- All the trigonometric symbols i.e., cosine, sine, tangent, cotangent, secant and cosecant, have no literal meaning.
- $(\sin \theta)^n$ is generally written as $\sin^n \theta$, n being a positive integer. Similarly, other trigonometric ratios can also be written.
- The values of the trigonometric ratios of an angle do not vary with the length of the sides of the triangle, if the angles remain the same.

5. Pythagoras theorem:

It states that “in a right triangle, square of the hypotenuse is equal to the sum of the squares of the other two sides”.

Pythagoras theorem can be used to obtain the length of the side of a right angled triangle when the other two sides are already given.

6. Relation between trigonometric ratios:

The ratios cosec A, sec A and cot A are the reciprocals of the ratios sin A, cos A and tan A respectively as given:

- $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$
- $\operatorname{sec} \theta = \frac{1}{\cos \theta}$
- $\tan \theta = \frac{\sin \theta}{\cos \theta}$
- $\cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$

7. Values of Trigonometric ratios of some specific angles:

$\angle A$	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

- The value of sin A or cos A never exceeds 1, whereas the value of sec A or cosec A is always greater than 1 or equal to 1.
- The value of $\sin \theta$ increases from 0 to 1 when θ increases from 0° to 90° .
- The value of $\cos \theta$ decreases from 1 to 0 when θ increases from 0° to 90° .
- If one of the sides and any other parts like either an acute angle or any side of a right triangle are known, the remaining sides and angles of the triangle can be obtained using trigonometric ratios.

8. Trigonometric ratios of complementary angles:

Two angles are said to be complementary angles if their sum is equal to 90° . Based on this relation, the trigonometric ratios of complementary angles are given as follows:

- i. $\sin (90^\circ - A) = \cos A$
- ii. $\cos (90^\circ - A) = \sin A$
- iii. $\tan (90^\circ - A) = \cot A$
- iv. $\cot (90^\circ - A) = \tan A$
- v. $\sec (90^\circ - A) = \operatorname{cosec} A$
- vi. $\operatorname{cosec} (90^\circ - A) = \sec A$

Note: $\tan 0^\circ = 0 = \cot 90^\circ$, $\sec 0^\circ = 1 = \operatorname{cosec} 90^\circ$, $\sec 90^\circ$, $\operatorname{cosec} 0^\circ$, $\tan 90^\circ$ and $\cot 0^\circ$ are not defined.

9. Definition of Trigonometric Identity

An equation involving trigonometric ratios of an angle, say θ , is termed as a **trigonometric identity** if it is satisfied by all values of θ .

10. Basic trigonometric identities

- i. $\sin^2 \theta + \cos^2 \theta = 1$
- ii. $1 + \tan^2 \theta = \sec^2 \theta; 0 \leq \theta < 90^\circ$
- iii. $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta; 0 \leq \theta < 90^\circ$